Section 15.5

The Directional Derivative

We would like to measure the instantaneous rate of change of $z = f(x, y)$ at (a, b) as (x, y) is restricted to a given line through (a, b).

Definition: Directional Derivative
Let f be differentiable at (a, b) and let $\vec{u} = \langle u_1, u_2 \rangle$ be a unit vector in the xy-plane. The **directional derivative of f at (a, b) in the direction of $\vec{u}$** is

$$D_{\vec{u}} f(a, b) = \lim_{h \to 0} \frac{f(a + hu_1, b + hu_2) - f(a, b)}{h}$$

How do we actually compute these derivatives?

Theorem 15.10: Directional Derivative
Let f be differentiable at (a, b) and let $\vec{u} = \langle u_1, u_2 \rangle$ be a unit vector in the xy-plane. The **directional derivative of f at (a, b) in the direction of $\vec{u}$** is

$$D_{\vec{u}} f(a, b) = \langle f_x(a, b), f_y(a, b) \rangle \cdot \langle u_1, u_2 \rangle$$

Example 15.5.1. Determine the directional derivative of

$$f(x, y) = \ln(2xy) - \sin(x^2 + y^2)$$

in the direction of $\vec{v} = \langle 7, -3 \rangle$.

Definition: Gradient
Let f be differentiable at the point (x, y). The **gradient** of f at (x, y) is the vector valued function

$$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle.$$

Example 15.5.2. Find the gradient of $f(x, y) = y^3x^5 + \ln(xy)$

For a function $f(x, y)$ and point $P(a, b)$, maximize $D_{\vec{u}}f(a, b)$ over all unit vectors $\vec{u}$.

Example 15.5.3. For the paraboloid $z = 4 + x^2 + 3y^2$, at the point $P(2, -\frac{1}{2}, \frac{35}{4})$ in what direction is the steepest descent?